IIJ Institutional Investor Journals

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The Journal of Alternative Investments

Fall 2016 – Volume 19 Number 2 | www.iiJAI.com

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In the last two decades, risk parity has become quite popular with institutional investors and is often used to construct both traditional and alternative investments portfolios (see Da Silva, Lee, and Pornrojnangkool [2008], Allen [2010], Levell [2010], Maillard, Roncalli, and Teiletche [2010], Lee [2011], and Asness, Frazzini, and Pederson [2012] for reviews of the approach and additional literature). The classic risk parity asset allocation starts from determining the expected volatilities and correlations of portfolio assets while ignoring their return expectations. Historical estimates for the volatility and correlation values are often used, with the results usually robust even for the small historical sample size.[1] This property makes risk parity particularly suitable for the construction of portfolios of hedge fund assets that are notorious for having a limited amount of historical data available for analysis.

Still, there are many who discard risk parity for a single reason: It explicitly disregards investor views on the level of asset valuations. One example that is often used to demonstrate drawbacks of the risk parity approach is allocation to long-term bonds in the current environment of low interest rates. Risk parity assigns an overweight to low-volatility bonds that, in the current interest rate environment, have much more room to go down in price than up, a dynamic with which many investors are not comfortable.

Another frequently used example is the manner in which risk parity allocates to credit-linked assets, including credit long–short and fixed income relative value hedge funds at the peaks and troughs of the business cycle. Credit-linked assets tend to exhibit their lowest levels of volatility at the peak of the business cycle when these assets are most overvalued. Conversely, at the lowest point of the business cycle, in the midst of the crisis, credit-linked assets tend to exhibit their highest level of volatility and, at the same time, to be undervalued. Classic risk parity, guided exclusively by volatility considerations, would overweight credit when it is most overvalued and, conversely, take capital away when credit presents the best value opportunity, precisely the opposite of what a prudent investor would prefer to do.

In the following sections, we reformulate a risk parity approach in a way that allows us to incorporate investor asset valuation views. The key change that we suggest is to switch from volatility to the expected maximum drawdown (EDD) as a principal measure of asset risk within the risk parity framework. As we will demonstrate, such a change makes incorporating investor views into the risk parity framework easy and intuitive, which in our opinion significantly enhances this already popular portfolio construction framework.

It is important to mention that several recent attempts to incorporate investor views into the risk parity framework have been made. Jurczenko and Teiletche [2015] observed that the risk parity portfolio coincides with the particular case of a mean–variance optimal portfolio—one in which individual Sharpe ratios of all assets, adjusted for their diversification characteristics, are equal. The authors then evaluated the expected returns of all assets implied by such an equal Sharpe ratio mean–variance framework and fused those returns with investor views utilizing the Black–Litterman approach. The resulting framework enables investors to gradually pivot from the purely risk-parity-based portfolio on one end to the purely mean–variance optimal portfolio with asset expected returns determined by investor views on the other. In somewhat related work, Haesen et al. [2014] employed the Black–Litterman technique with risk parity allocation used as a prior for an otherwise mean–variance optimal portfolio. Also, Medvedev [2015] showed that one can interpret risk parity as an optimal mean–variance allocation when expected returns are ambiguous and discussed how investor views on, for example, rank of assets' returns may be incorporated into such a mean–variance optimization framework. All of these approaches are in some way rooted in the mean–variance optimization framework, which fundamentally attempts to find portfolios with the highest level of return for a given level of volatility.

Another body of relevant work addresses the problem of portfolio return maximization with a drawdown constraint. It has been demonstrated (see, e.g., Chekhlov, Uryasev, and Zabarankin [2005] and Davidsson [2012]) that such an optimization problem can be reduced to a linear programming problem, which makes it computationally feasible for any number of assets. To the best of our knowledge, this work discussed neither the risk parity framework with any sort of drawdown constraint nor how investor views can be incorporated into such framework.

We, on the other hand, see much benefit in creating a risk parity framework that fundamentally focuses on drawdowns as the measure of risk. We will also demonstrate that such a framework allows for incorporating investor views quite efficiently and with welcome clarity.

RISK PARITY AND EXPECTED DRAWDOWNS

Before formulating the EDD-based risk parity framework, let us briefly walk through what we will call *classic* volatility- and correlations-based risk parity. Here, we largely follow Lee [2011].

Classic risk parity stipulates that all assets in a portfolio should be allocated to in such a way that the percentage contribution of each of these assets to portfolio volatility must be the same:

$$w_i \frac{\partial \sigma_P}{\partial w_i} \frac{1}{\sigma_P} = \frac{1}{N} \tag{1}$$

Here, N is the number of assets in the portfolio (which makes $1/N$ the *risk budget* assigned by risk parity to each asset), σ_P is the portfolio volatility, and w_i is the weight of the i-th asset. Taking into account that portfolio volatility is easily expressed in terms of asset weights w_i, volatilities σ_i, and covariances σ_{ij}, Equation (1) translates into a system of equations for the risk parity asset weights:

$$w_i \frac{1}{\sigma_P{}^2} \sum_j w_j \sigma_{ij} = \frac{1}{N} \tag{2}$$

This equation is easy to solve numerically for the arbitrary values of asset volatilities and covariance coefficients. In the simple case of perfectly uncorrelated assets, $\sigma_{i \neq j} = 0$ and $\sigma_{ii} = \sigma_i^2$, the resulting risk parity weights are inversely proportional to asset volatilities, a well-known classic result:

$$w_i = \frac{1}{\sigma_i} \frac{1}{\sum_j \left(1/\sigma_j \right)} \tag{3}$$

Let us now reformulate risk parity using EDD as a measure of risk. We define EDD as the expected value for the maximum amount of money an asset can lose, peak to trough in percentage terms, over a predefined period of time. In the spirit of Equation (1), we stipulate that all assets in a portfolio must be allocated in such a way that the percentage contribution of each asset to the portfolio expected drawdown is the same:

$$w_i \frac{\partial EDD_P}{\partial w_i} \frac{1}{EDD_P} = \frac{1}{N} \tag{4}$$

This equation looks very similar to Equation (1). Furthermore, for the realistic case of average returns on assets being much smaller than asset volatilities (which happens to be the case for the majority of financial assets), expected drawdown and volatility are linked by an approximately linear relationship (see Magdon-Ismail et al. [2004]):

$$EDD(T) = -\alpha_{Vol}\ \sigma\ \sqrt{T}. \tag{5}$$

Here, T is the period of time over which the drawdowns are measured, and $\alpha_{Vol} \approx 1.07$ for the log-normally distributed asset prices.[2] The minus sign on the right-hand side of Equation (5) reflects the fact that the drawdown always represents a negative return.

At first glance and given the direct linear relationship between expected drawdown and volatility, Equation (4) seems to be practically identical to the original risk parity definition, Equation (1). There is, however, a significant difference. Asset volatility is agnostic to the overall direction of asset moves and is only dependent on the size and timing of those moves. EDD, however, is by definition a directional measure that reflects an asset price's potential downside. This property of the expected drawdown, combined with Equation (4), allows us to incorporate investor views on future asset valuations into the risk-parity-based portfolio construction framework.

INVESTOR VIEWS AND THE DRAWDOWN-BASED RISK PARITY

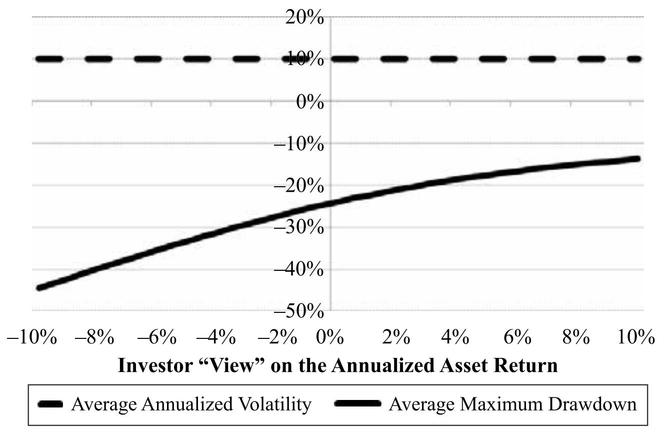
For the purposes of this study, we define *investor views* as shifts in asset prices forecasted by the investor over a given period of time. In the case of a single asset, such views on, say, the monthly returns of such an asset may be expressed in a form of a simple equation:

$$r(t_i) = r^{NV}(t_i) + \Delta p \tag{6}$$

where $r(t_i)$ and $r^{NV}(t_i)$ are the monthly returns of this asset in the month t_i and in the absence of views (*no views* [*NV*]). In turn, Δp is the monthly return forecast provided by the investor or, in other words, the view itself. A simulation may be utilized to study how the magnitude of such a view affects both the volatility and expected drawdown of the asset. Exhibit 1 presents an example of such a study. We started with an asset whose no-view returns are normally distributed with the expected return of zero and an annualized standard deviation of 10%. A Monte Carlo simulation of returns of such an asset over a 5-year period shows an expected maximum

EXHIBIT 1

Average Asset Volatility and Average Maximum Drawdown in the Presence of Investor Views



drawdown of around 25%. We subsequently applied the view Δp that ranged between −10% and +10% per year and ran the simulation again over the same period. As one can see from Exhibit 1, positive views on asset valuation result in reduced (but not eliminated) average drawdowns, and conversely, negative views on asset valuation result in the average drawdowns becoming deeper. Not surprisingly, investor views have no effect whatsoever on the volatility of asset returns.

Exhibit 1 illuminates why an EDD-based risk parity framework is better suited to incorporating investor views than the classic one. As demonstrated, such views have no effect on asset volatility and hence on classic risk parity results. In contrast to this, views affect the expected drawdown, a dependence that we can approximate as

$$EDD(T) = -\alpha_{Vol}\sigma\sqrt{T} + \alpha_{view} T\ \Delta p \tag{7}$$

The first term of Equation (7) represents drawdown expectation over time T in the absence of investor views, while the second term (our simulation yields $\alpha_{view} \approx 0.33$) represents the effect of investor views. Equation (7) incorporates views into the EDD formula as a linear add-on. This is a simplification but a good one as long as the first term in Equation (7) dominates. Drawdown's dependence on a view is approximated reasonably well by a linear function for a rather wide range of such views.

The direct effect investor views have on the expected drawdown will, in turn, allow such views to be incorporated into the EDD-based risk parity asset allocation. Let us illustrate this with a simple example. Say, for example, we are trying to use the risk parity approach to determine weights of a two-asset portfolio composed of equities, as represented by the S&P 500 Total Return index and U.S. bonds, as represented by the Barclays U.S. Aggregate Bond index. In the absence of investor views, regardless of whether classic or EDD-based risk parity is used, those weights will be based exclusively on the differences in risk of these assets, giving low-risk bonds a significant overweight. See Exhibit 2.

Investor views—applied through the EDD-based risk parity framework of Equations (4) and (7)—may significantly influence those results. Let us, for the sake of this example, assume that we have no views on the behavior of equity markets but do believe that the government bond market will experience a significant 10% price deterioration as interest rates rise over the next 12 months. Given such a view, a prudent investor would want to reallocate some capital from bonds to equities; the EDD-based risk parity framework does precisely that. Please see Exhibit 2 for the summary of results.

As follows from Exhibit 2, negative investor views on the fixed income markets led to an increase in equity allocation at the expense of bonds. This reallocation is moderate given how volatile equities are as compared to the bonds in this example.

INVESTOR VIEWS ON BROAD MARKET FACTORS AND RISK PARITY

To make the new, EDD-based risk parity framework better suited to the needs of institutional investors, we would like to take one more step. More often than not, investors have well-developed views on the broad markets—national or global stock indexes, credit spreads, sovereign interest rates, inflation, and so on—as opposed to views on specific securities. It is therefore helpful to reformulate the risk parity framework—both classic and EDD-based—in terms of factor exposures to those broad markets.

If one assumes that there are factors that are common across multiple assets, then the returns of these assets may be expressed through a simple linear model:

$$r_i(t) = \varepsilon_i(t) + \sum_m \beta^i_m \Delta F_m(t) \quad (8)$$

where ε_i is uncorrelated random variables with volatilities of $\langle \varepsilon_i^2 \rangle = (\sigma_i^\varepsilon)^2$. Notably, covariances between assets now come only through covariances between factors:

$$\sigma_{ij}^{NV} = \langle r_i^{NV} r_j^{NV} \rangle = \left(\sigma_i^\varepsilon\right)^2 \delta_{ij} + \sum_{mn} \beta^i_m \beta^j_n \lambda_{mn} \quad (9)$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise, while λ_{mn} is the interfactor covariance.

The EDD-based risk parity framework in the presence of investor views on factor returns can also be formulated, starting with the drawdown estimation itself:

$$EDD_i(T) = -\alpha_{Vol}\ \sigma_i^{NV}\sqrt{T} + \alpha_{View} T \sum_m \beta^i_m \Delta F_m \quad (10)$$

The first term represents the drawdown expectation in the absence of investor views (σ_i^{NV} is asset [or portfolio] volatility), while the second term represents the effect of investor views (ΔF_m is views on factor *m*

EXHIBIT 2
EDD-Based Risk Parity Framework, 10% Price Deterioration in Bond Market

Portfolio Component	Index Used for Analysis	Annualized Volatility	Beta to Citibank U.S. Treasury 10-Year Index	No Investor Views: Classic Risk Parity Allocations	No Investor Views: EDD-Based Risk Parity Allocations	Investor Annualized Views: Citibank U.S. Treasury 10-Year Index: –10%: Classic Risk Parity Allocations	Investor Annualized Views: Citibank U.S. Treasury 10-Year Index: –10%: EDD-Based Risk Parity Allocations
Equities	S&P 500 Total Return Index	14.6%	0	20%	20%	20%	24%
Fixed Income	Barclays Aggregate Bond Index	3.7%	0.46	80%	80%	80%	76%
Total				**100%**	**100%**	**100%**	**100%**

over the time horizon T per unit of time). One may view Equation (10) as an illustration of a well-known expression that "a rising tide lifts all boats." If an investor believes that the broad market goes up (down) by $X\%$ during a time period T, said investor should assume that drawdowns of all assets in this market will (on average and after beta adjustment) decrease (increase) proportionally to $X\%$ as compared to the no-views scenario. The precise coefficient of this proportionality—α_{View}—is a function of the drawdown definition (in our case simulation shows $\alpha_{View} \approx 0.33$), but this dynamic will hold for all drawdown-type measures.

We can now combine Equation (4), which defines our EDD-based risk parity framework, with Equation (10), that incorporates views into the EDD statistic to arrive at the expression representing the new framework incorporating factor-based investor views:

$$w_i \left[\frac{1}{(\sigma_P^{NV})^2} \sum_j w_j \sigma_{ij}^{NV} - \Omega_i \right] \frac{1}{1 - \Omega_P} = \frac{1}{N}$$
$$\Omega_i = \alpha \sqrt{T} \left(\sum_m \beta_m^i \Delta F_m \big/ \sigma_P^{NV} \right)$$
$$\Omega_P = \sum_i w_i \Omega_i \tag{11}$$

Here, Ω_i and Ω_P are normalized views for the asset i and for the portfolio as a whole, $\alpha = \alpha_{View}/\alpha_{Vol} \approx 0.31$, and historical data can be used to evaluate σ_P^{NV}, σ_{ij}^{NV}, and β_m^i.

Equation (11) is the central result of this article. It was derived under the realistic assumption that the contribution of investor views into the future asset returns are small enough to warrant the linear representation used in Equation (10), or in other words, both Ω_P and Ω_i are smaller than 1. Equation (11) incorporates investor views on asset prices into the quintessentially risk-parity-based asset allocation framework. In the case when no investor views exist, $\Omega_i = \Omega_P = 0$, Equation (11) becomes identical to the classic risk parity result of Equation (2).

While Equation (11) may seem cumbersome, its application for portfolio construction is quite straightforward. Factor formulation is especially helpful when assets in the portfolio are complex and difficult to predict individually and while certain commonalities between those complex assets exist, are persistent, and are well understood. Hedge funds, in general, and liquid alternative investments, in particular, are good examples of such assets and deserve a separate discussion.

HEDGE FUND PORTFOLIO MANAGEMENT AND EDD-BASED RISK PARITY

Hedge fund portfolio management presents a number of unique challenges to institutional investors. In addition to the issue of illiquidity—something that is somewhat remedied in the case of liquid alternatives—hedge fund portfolio assets are dynamic and complex, each on a look-through basis representing a portfolio of securities in its own right. Furthermore, most hedge fund strategies—liquid or not—typically do not have the luxury of decades of historical performance available for the investor. Instead, the average lifespan of a hedge fund is about 5 years. Such a small sample size, combined with the inherently dynamic nature of hedge fund strategies, makes predicting returns of individual hedge funds over a reasonable investment horizon (say, 12 months) extremely challenging and an imprecise science (see, for example, Michaud [1989]). Consequently, many institutional investors have switched from using mean–variance optimization that requires such predictions as inputs to a number of alternative approaches to hedge fund portfolio construction, including pure discretion, diversification-based allocations (see Rudin and Morgan [2006], Choueifaty and Coignard [2008], and Crezee and Swinkels [2010]) and, in many cases, risk parity.

We believe that the modified, drawdown-based risk parity approach introduced in this article is well suited for hedge fund portfolio management. It is as robust as the classic risk parity and, hence, suitable for the small historical sample sizes. Also, while it is difficult to predict idiosyncratic behavior of a given hedge fund, it is well established by both academics and practitioners (see, e.g., Bussiere, Hoerova, and Klaus [2014] for the review of relevant literature) that many hedge fund strategies share common factors or exposures to various risk premiums, from simple ones like the stock market index or the level of nominal interest rates to more complex ones like currency carry or trend.

It is worth noting that simple commonality of a factor across multiple hedge funds or hedge fund strategies is not sufficient for the fruitful inclusion of this factor into Equation (11). The factor should also be such that investors may form views on it with a reasonable degree of confidence. For example, both economists and institutional investors often form views on the levels of stock markets, currency exchange rates, inflation, interest rates, and so on. On the other hand, forming

EXHIBIT 3

Asset Allocation Results before and after 12-Month Elevated Risk Aversion in Global Financial Market with Equity Market Correction and U.S. Government Bond Market Rally

Portfolio Component	Index Used for Analysis	Annualized Volatility	Beta to S&P 500 Total Return Index	Beta to Citibank U.S. Treasury 10-Year Index	No Investor Views: Classic Risk Parity Allocations	No Investor Views: EDD-Based Risk Parity Allocations	Investor Views: S&P 500 Total Return Index: –20% Citibank U.S. Treasury 10-Year Index: +10%: Classic Risk Parity Allocations	Investor Views: S&P 500 Total Return Index: –20% Citibank U.S. Treasury 10-Year Index: +10%: EDD-Based Risk Parity Allocations
Equity Long–Short	HFRI Equity Hedge Total Index	9.0%	0.4	0.0	27%	27%	27%	24%
Credit Long–Short	HFRI Event Driven Credit Arbitrage Index	6.9%	0.3	0.0	38%	38%	38%	36%
Managed Futures	Newedge Commodity Trading Advisor Index	8.7%	0.0	0.2	34%	34%	34%	40%
Total					**100%**	**100%**	**100%**	**100%**

views on the future performance of, say, the implied/realized volatility spread, is much more challenging.

Let us now illustrate how one could incorporate investor views into the risk-parity-based alternative asset allocation scheme using the example of a classic hedge fund portfolio consisting of an equity long–short component, a credit long–short component, and a diversifier such as managed futures. While it is difficult to predict the behavior of those components, per se, we may have views on the future performance of the underlying equity and bond markets. The idea expressed in this article is to use such views for tactical asset allocation. Let us say we have a view that risk aversion in the global financial market will be elevated over the next 12 months, resulting in equity markets, on average, exhibiting a correction and U.S. government bond markets rallying at the same time. Exhibit 3 illustrates asset allocation results before and after applying those views.

The results follow an intuitive pattern. In the absence of investor views, classic and EDD-based approaches allocated to the three portfolio components in the same way. Applying views yields no changes to the classic, volatility-based approach. On the other hand, negative views on the equity market combined with constructive views on bonds leads to reallocation of capital toward assets expected to have lower drawdowns in the predicted environment, in this case managed futures. The degree of reallocation is substantive but not dramatic in this case, reflecting a quantitatively discovered balance between the strength of investors' discretionary views on markets, portfolio assets' sensitivity to those views, and the amount of unpredictable, idiosyncratic risk within the portfolio determined by the level of asset volatility.

CONCLUSION

In this article, we introduce a methodology to inform a risk-based asset allocation framework, such as risk parity, of investor views on asset prices or on factors that drive prices. The main idea is to switch from volatility to the expected drawdown as a principal measure of asset risk within the risk parity framework. As the drawdown measure is directly dependent on investor views on future asset valuations, such a change makes incorporating these views into the risk parity framework possible and intuitive. We believe that the proposed enhancement may significantly improve the risk parity framework, broaden its appeal with the institutional investor community, and be particularly helpful for the hedge fund portfolio construction.

ENDNOTES

Alexander Rudin would like to express gratitude to Anna Gapon for illuminating discussions contributing to this article.

[1]An illuminating study of relative robustness of mean–variance optimized and risk parity portfolio construction

approaches applied to the same simulated data samples was performed by Rappoport and Nottebohm [2012]. The study convincingly demonstrated advantages of risk parity for small historical sample sizes and for situations in which expected Sharpe ratios of the portfolio assets are not massively different.

[2]A slight difference between the numerical coefficient in Equation (5) and results described by Magdon-Ismail et al. [2004] is related to the fact that their article considered arithmetic Brownian motion, while we used a more applicable geometric Brownian motion to simulate returns.

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